



Real options models

(step-by-step examples of solving real options models)

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Option to abandon



Suppose a pharmaceutical company is developing a new drug. Due to the uncertain nature of the drug's development, market demand, success in human and animal testing, and FDA approval, **management has decided that it will create a strategic abandonment option.**

If the program is terminated, **the firm can potentially sell off its intellectual property rights** of the drug to another pharmaceutical firm with which has a contractual agreement. **This contract is exercisable at any time within the next five years.** After five years, the firm would have either succeeded or completely failed in its drug development, so no option value after that time period.

Option to abandon



Using a traditional DCF model, the present value of the expected future cash flows is \$ 150 million. Using Monte Carlo simulation, the implied volatility is 30%. The risk free rate for the same time frame is 5% and patent is worth \$100 million if sold within the next five years. Assume that this \$ 100 million salvage value is fixed for the next five years.

You attempt to calculate how much the abandonment option is worth and if the efforts to develop the drug is worth to the firm.

Option to abandon



Using the Bierksund closed-form American put option you calculate the value of the option to abandon as \$6,9756 million.

Using the binomial approach, the value is \$6,55 million using 5 time-steps and \$7,0878 million using 1.000 time-steps.

Option to abandon



u 1,34986
 d 0,74082

 p 0,512
 $1-p$ 0,488

	0	1	2	3	4	5
0	150	202,48	273	369	498	672
1		111	150	202	273	369
2			82	111	150	202
3				61	82	111
4					45	61
5						33
6						
7						

	0	1	2	3	4	5
0	6,55	2	0	0	{ 0,00 0,00 0 0,00 0 0,00 18 0,00 55 39,01 66,53	
1		12	4	0		
2		0	22	8		
3			0	39		
4				0		
5						
6						
7						

← The nodes at the end of the lattice are valued first, going from right to the left.

Second, the intermediates nodes are valued using a process called "backward induction"

Option to abandon



At the end of five years, **the firm has the option to both sell off and abandon the drug program or to continue developing.**

The value of abandoning the drug program is \$100 million, equivalent to sell the patent rights.

The value of continuing with development is the lattice evolution of underlying asset ($S_0u^5=672,2$ million in node) or $S_0d^5=33$ million in node)

Option to abandon



	0	1	2	3	4	5
0	150	202,48	273	369	498	672
1		111	150	202	273	369
2			82	111	150	202
3				61	82	111
4					45	61
5						33

[Go to the spreadsheet](#)

If the underlying asset value of pursuing the drug development is high (node A) it is wise to continue with the development. But if the value of the development down to such a **low level like the lower branches of the tree, then it is better to abandon the project and cut the firm's losses.**

Using the backward induction technique and back to the starting point we obtain the value of \$156,55 million. Because the value obtained using DCF is \$150 million, the difference of \$6,55 million additional value is due to the abandonment option.

Option to expand



Suppose a growth firm has a static DCF value of \$400 million. Using Monte Carlo simulation you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 35%. The risk free rate is found yielding 7%.

Suppose **the firm has the option to expand and double its operations by acquiring its competitor for a sum of \$ 250 million at any time over the next five years.**

What is the **total value of the firm** considering the option to expand?

Option to expand



	0	1	2	3	4	5
0	400	567,63	805,50	1143,06	1622,08	2301,84
1		281,88	400,00	567,63	805,50	1143,06
2			198,63	281,88	400,00	567,63
3				139,98	198,63	281,88
4					98,64	139,98
5						69,51

	0	1	2	3	4	5
0	645,86	959,90	1418,32	2078,52	3018,14	4353,68
1		405,72	611,85	922,25	1381,15	2036,12
2		0,00	245,47	370,67	568,24	885,25
3			0,00	148,01	214,44	313,75
4				0,00	98,87	139,98
5						69,51

Using a binomial approach you calculate the value of the expansion option as \$645,86 million using 5 time-steps and \$638,8 using 1.000 time-steps.

Option to contract



You work for a large manufacturing firm that is unsure of the technological efficacy and market demand of its new product. The firm decides to hedge itself by using a strategic options, **contracting 50% of its manufacturing facilities at any time within the next five years**, thereby creating an additional \$400 million in savings after this contraction (the firm can scale back its existing work force to obtain this savings)

The present value of the expected cash flows is 1 billion. Using the Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on projected future cash flows to be 50%. The risk free rate is 5%.

Option to contract



u	1,6487
d	0,6065
p	0,427
$1-p$	0,573

	0	1	2	3	4	5
0	1000	1648,72	2718,28	4481,69	7389,05	12182,48
1		606,53	1000,00	1648,72	2718,28	4481,69
2			367,88	606,53	1000,00	1648,72
3				223,13	367,88	606,53
4					135,34	223,13
5						82,09

	0	1	2	3	4	5
0	1.111,59	1711,00	2743,91	4492,54	7398,00	12182,48
1		762,35	1091,34	1681,55	2721,57	4481,69
2			583,94	747,18	1054,02	1648,72
3				511,57	583,94	703,27
4					467,67	511,57
5						441,04

The real option value is worth an additional 11% or \$111,5 of existing business operations.

Different options can exist simultaneously



To modify the business case and make it more in line with actual business conditions, different options type can be accounted for at once (**chooser options**) or in phases (**compound options**). These options can exist **simultaneously** in time or come into being in **sequence** over a much longer period.

Compound options – drug development



A compound option is an option whose value depends on the value of another option.

For instance, a pharmaceutical company going through a FDA drug approval process has to go through human trials.

The success of the **FDA approval** depends on the success of **human testing**, both occurring at the same time.

The former costs \$900 million and the latter \$500 million. Both phases occur simultaneously and take three years to complete. The static valuation of the drug development effort's using a DCF model is found to be \$1 billion.

Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 30%. The risk free rate is 7,7%.

Compound options – drug development



u	1,3499
d	0,7408
p	0,557
$1-p$	0,443

Lattice Evolution of the Underlying Asset

	0	1	2	3
0	1000	1349,86	1822,12	2459,60
1		740,82	1000,00	1349,86
2			548,81	740,82
3				406,57

First step: lattice of the underlying asset, based on the up and down factors

Equity Lattice

	0	1	2	3
0	364,19	608,52	991,61	1559,60
1		120,32	232,65	449,86
2			0,00	0,00
3				0,00

Second step: calculation of the equity lattice, using risk neutral probabilities and the backward induction technique

Option Valuation Lattice

	0	1	2	3
0	146,56	283,40	547,98	1059,60
1		0,00	0,00	0
2			0,00	0
3				0

Third step: calculate the option valuation lattice.

The value of compound option is \$546,56 million (NPV with flexibility minus NPV without flexibility) notice how this compares to a static decision value of \$1.000-\$900-500=-\$400 million for the first investment.

Sequential Compound Options



A sequential compound option exists when a project has **multiple phases and latter phases depend on the success of previous phases.**

Suppose a project has two phases, where the first phase has a one-year expiration that costs \$500 million. The second phase's expiration is three years and costs \$700 million.

Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on projected future cash flows as 20%. The risk free rate is 7,7%.

The static valuation using a DCF model is found to be \$1 billion.

Sequential Compound Options



Lattice Evolution of the Underlying Asset

	0	1	2	3
0	1000	1221,40	1491,82	1822,12
1		818,73	1000,00	1221,40
2			670,32	818,73
3				548,81

Equity Lattice

	0	1	2	3
0	453,40	624,83	846,09	1122,12
1		235,94	352,87	521,40
2			71,54	118,73
3				0,00

Option Valuation Lattice

	0	1	2	3
0	75,21	124,83		
1		0,00		

First step: lattice of the underlying asset, based on the up and down factors

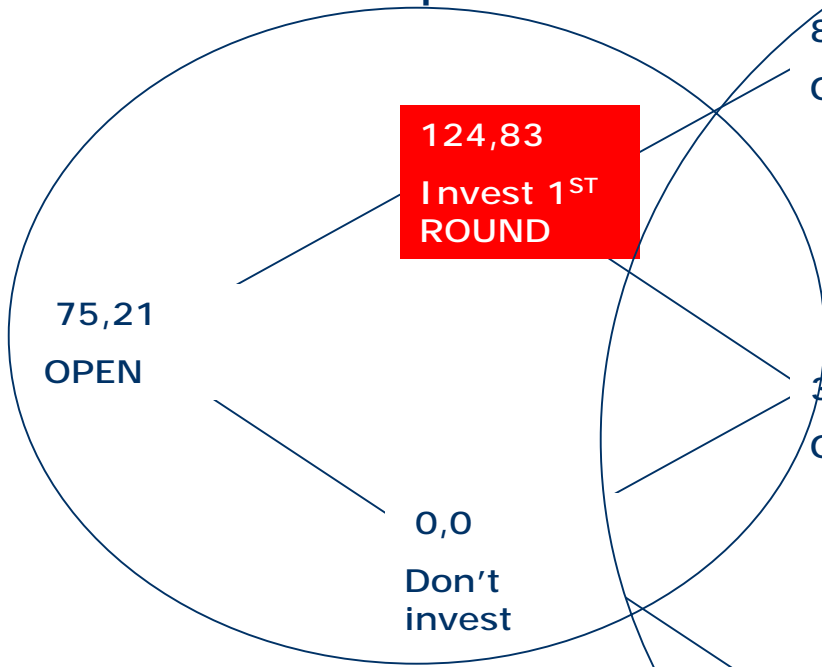
Second step: calculate the second, long-term option, using risk neutral probabilities and the backward induction technique

Third step: calculate the option valuation lattice. The analysis depends on the lattice of the second, long-term option.

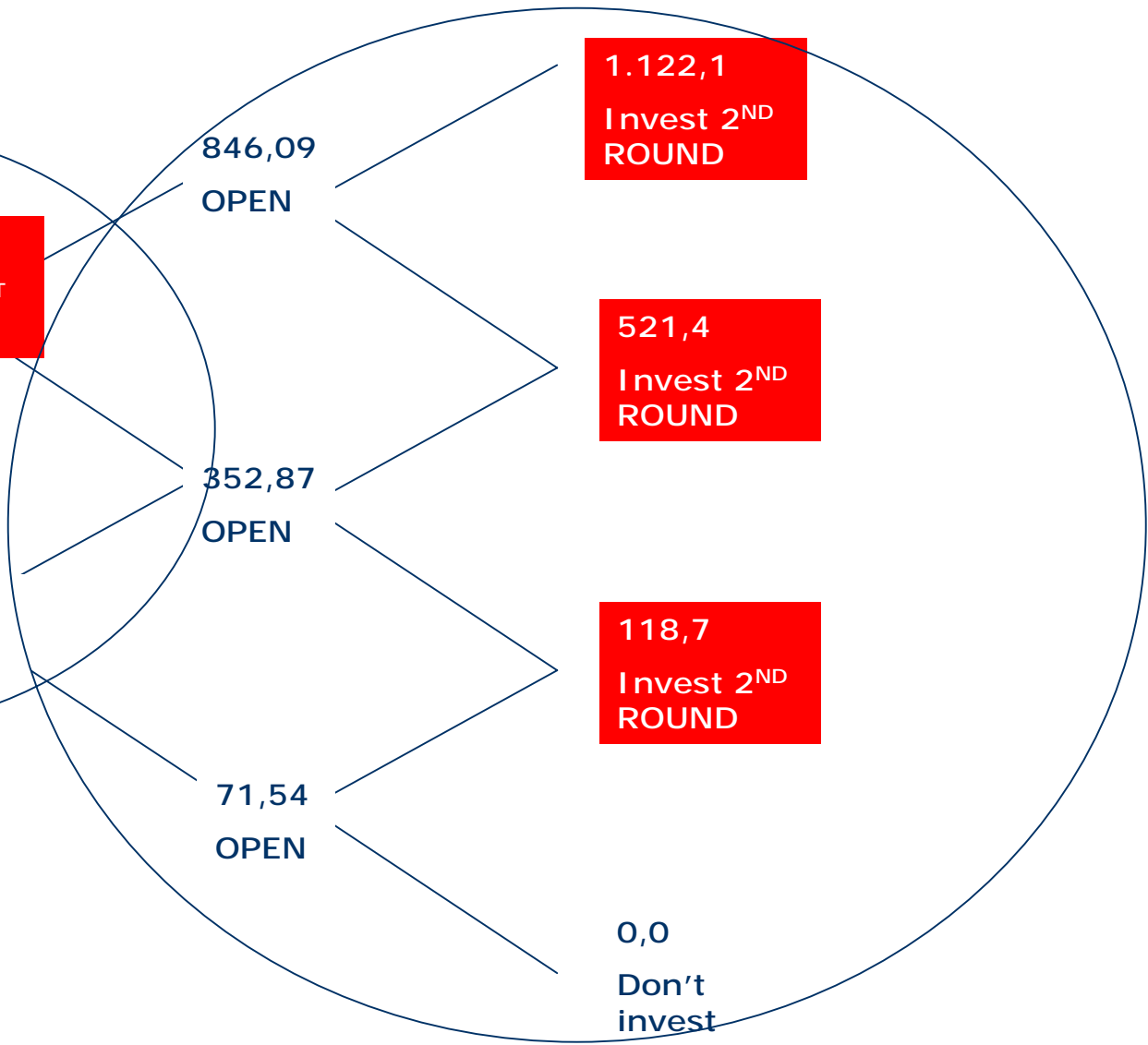
Sequential Compound Options



First option



Second option



Changing strikes



Sometimes, the implementation costs of the projects change. Suppose the implementation of a project in the first year costs \$80 million but increases to \$90 million in the second year due to expected increases in raw materials and input costs.

Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 50%. The risk free rate is 7%.

The static valuation using a DCF model is found to be \$1 billion.

Changing strikes



	0	1	2	
0	100	164,87	271,83	
1		60,65	100,00	
2			36,79	
	0	1	2	
0	37,63	84,87	181,83	← 271,83-90
1		4,18	10,00	← 100-90
2			0,00	

$$\text{MAX}[p181,83 + (1-p)10 / (1+rf); 164,87-80]$$

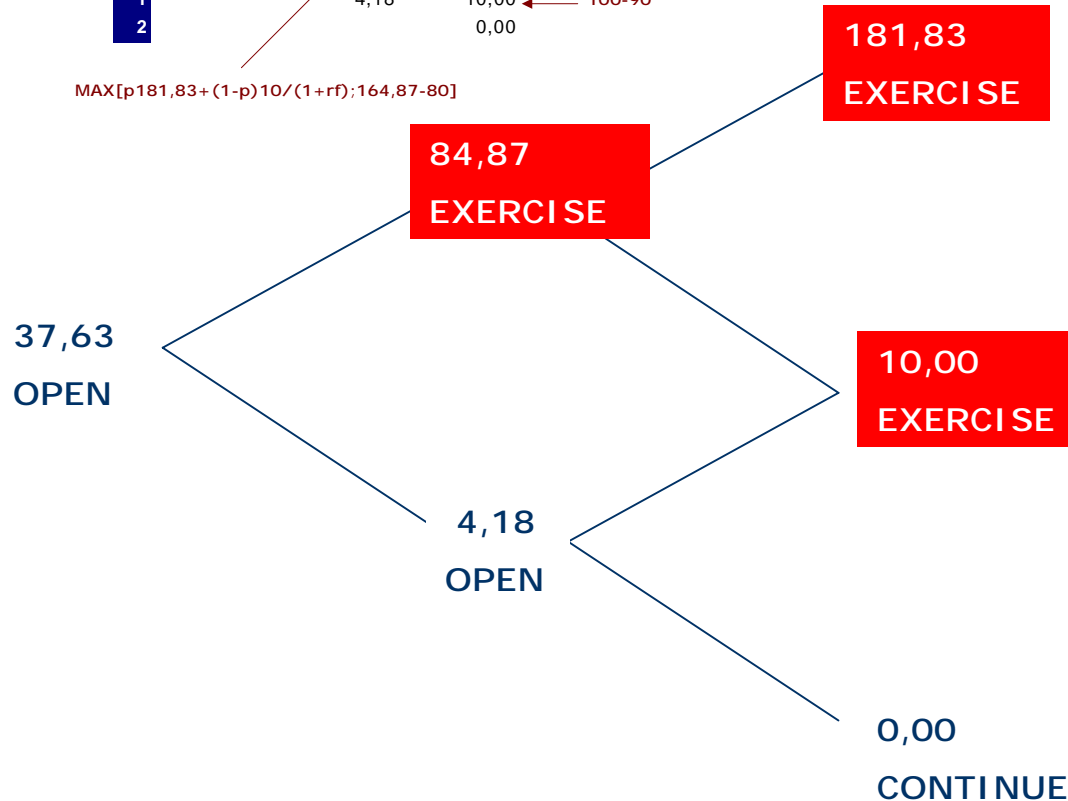
Changing strikes



	0	1	2
0	100	164,87	271,83
1		60,65	100,00
2			36,79

	0	1	2
0	37,63	84,87	181,83
1		4,18	10,00
2			0,00

$$\text{MAX}[p181,83 + (1-p)10 / (1+rf); 164,87 - 80]$$

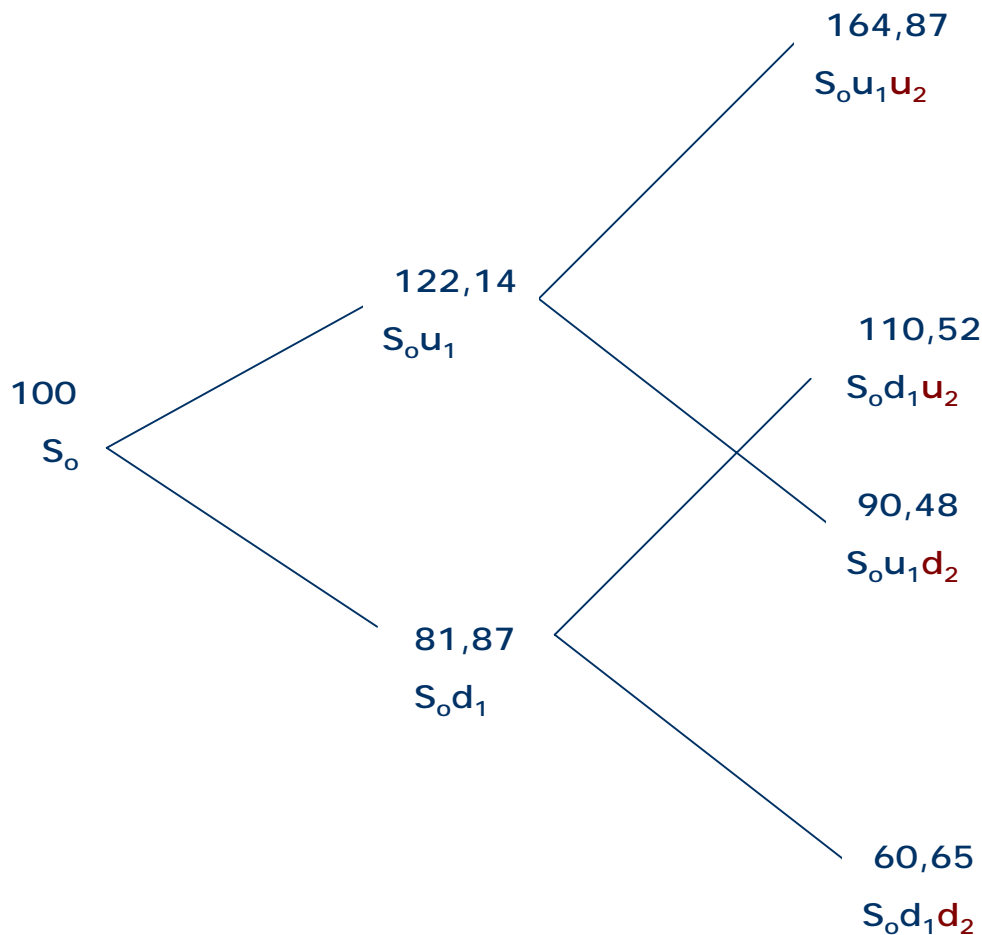


Notice that the value of the call option on changing strikes is \$37,63 million.

Compare this to a naive static DCF of \$20 million for the first year (100-80) and \$10 million (100-90) for the second year.

In actual business conditions, multiple strike costs can be accounted for over many time periods, and can also be used in conjunction with all other types of real options (expansion, compound, etc.)

Changing volatility

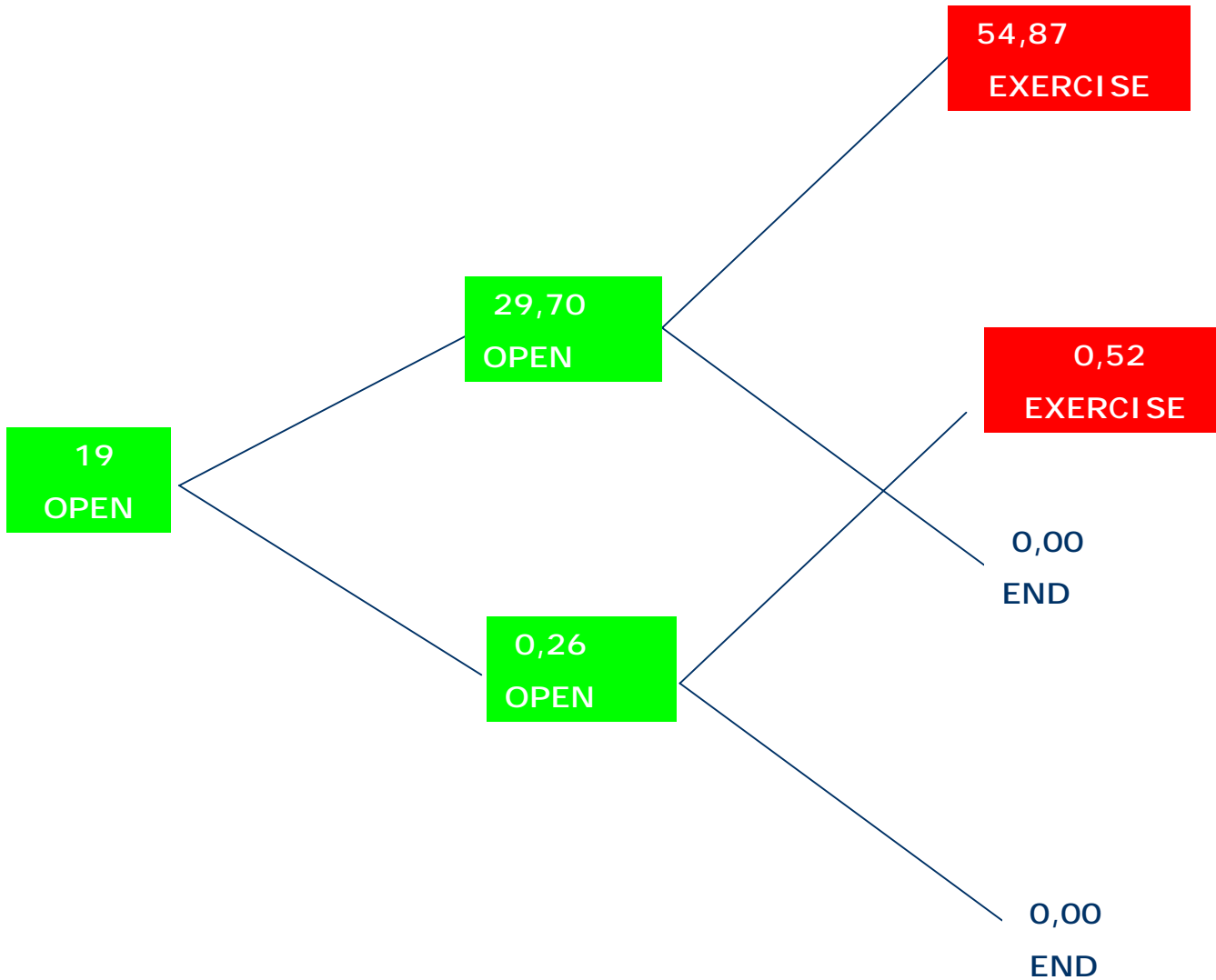


Instead of changing strike costs over time, volatility on cash flow returns may differ over time. Suppose a two year options where volatility is 20% in the first year and 30% in the second year.

In this circumstance, the up and down factor are different over the two time periods. Thus, the binomial lattice will no longer be recombining.

Strike price = 110

Changing volatility



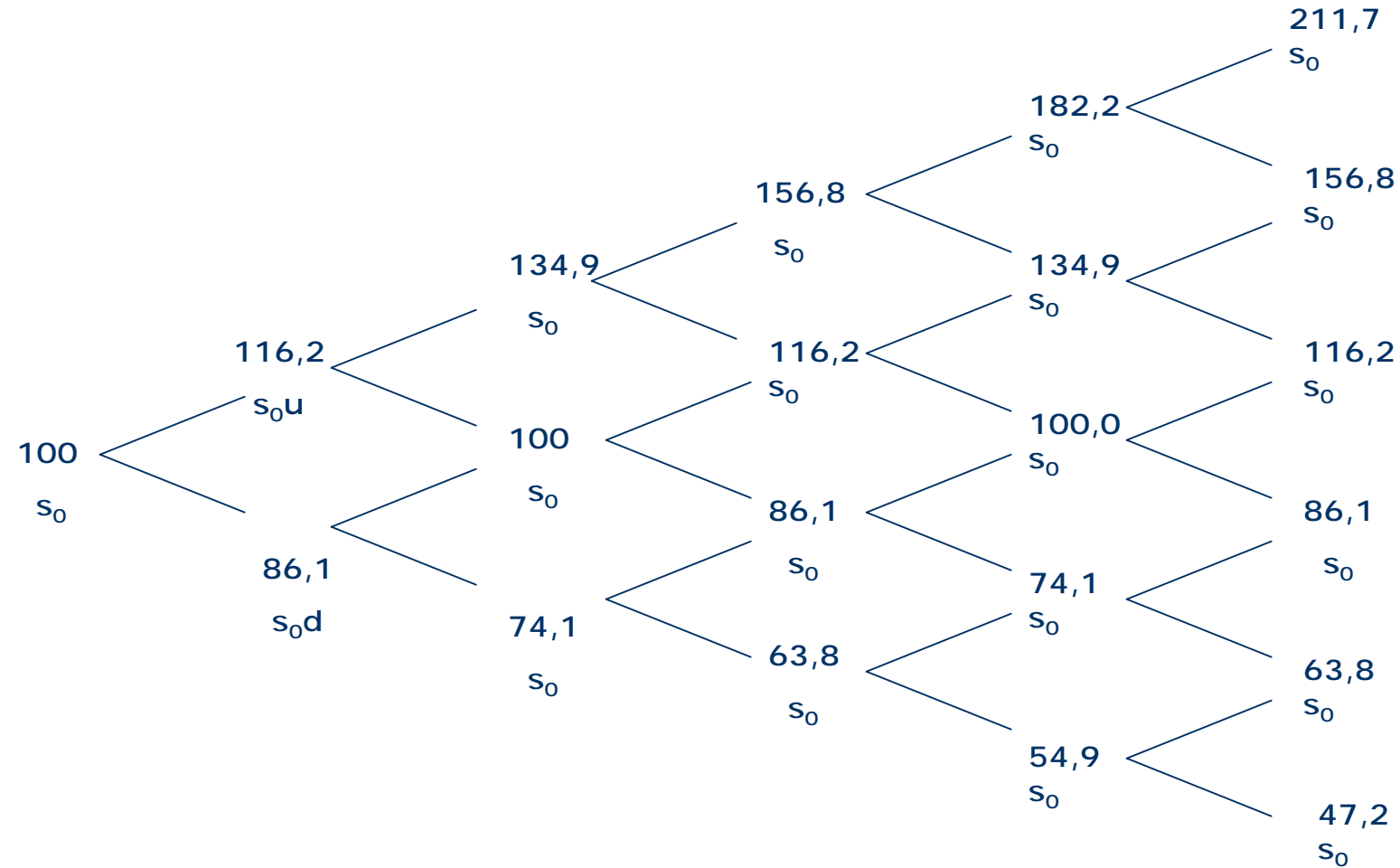
Option to choose



Suppose a large company decides to hedge itself through the use of strategic options. **It has the option to choose among three strategies: expanding or contracting its current operations and completely abandoning its business at any time within the next five years.** The static valuation of the current operating structure using a DCF model is found to be \$100 million. Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 15%. The risk free rate is 5%. The firm has the following options:

1. Contract 10% of its current operations, creating an additional \$25 million in savings after this contraction.
2. Expanding its current operations, increasing its value by 30% with a \$20 million implementation cost.
3. Abandoning its operations, selling its intellectual property for \$ 100 million

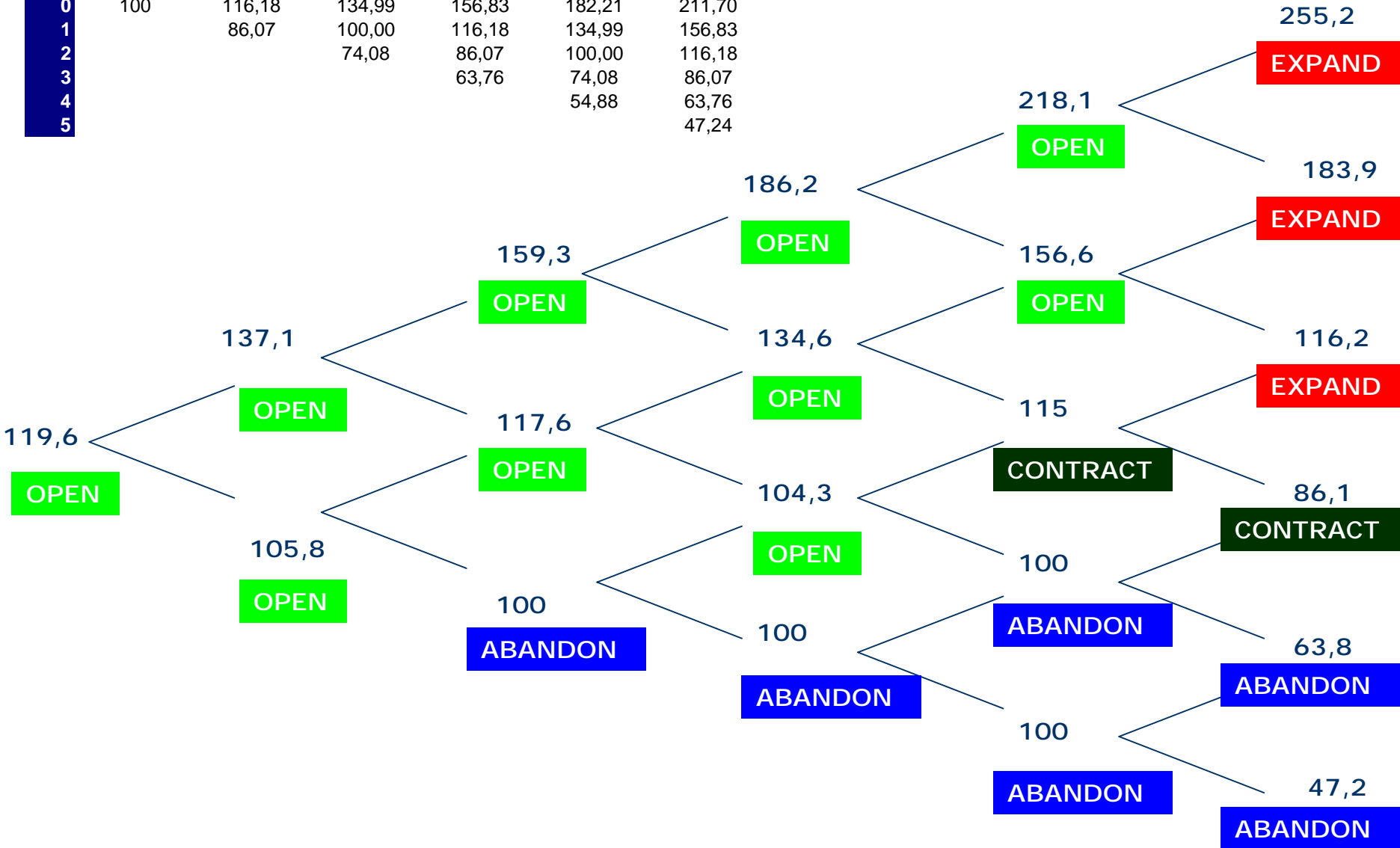
Option to choose



Option to choose



	0	1	2	3	4	5
0	100	116,18	134,99	156,83	182,21	211,70
1		86,07	100,00	116,18	134,99	156,83
2			74,08	86,07	100,00	116,18
3				63,76	74,08	86,07
4					54,88	63,76
5						47,24



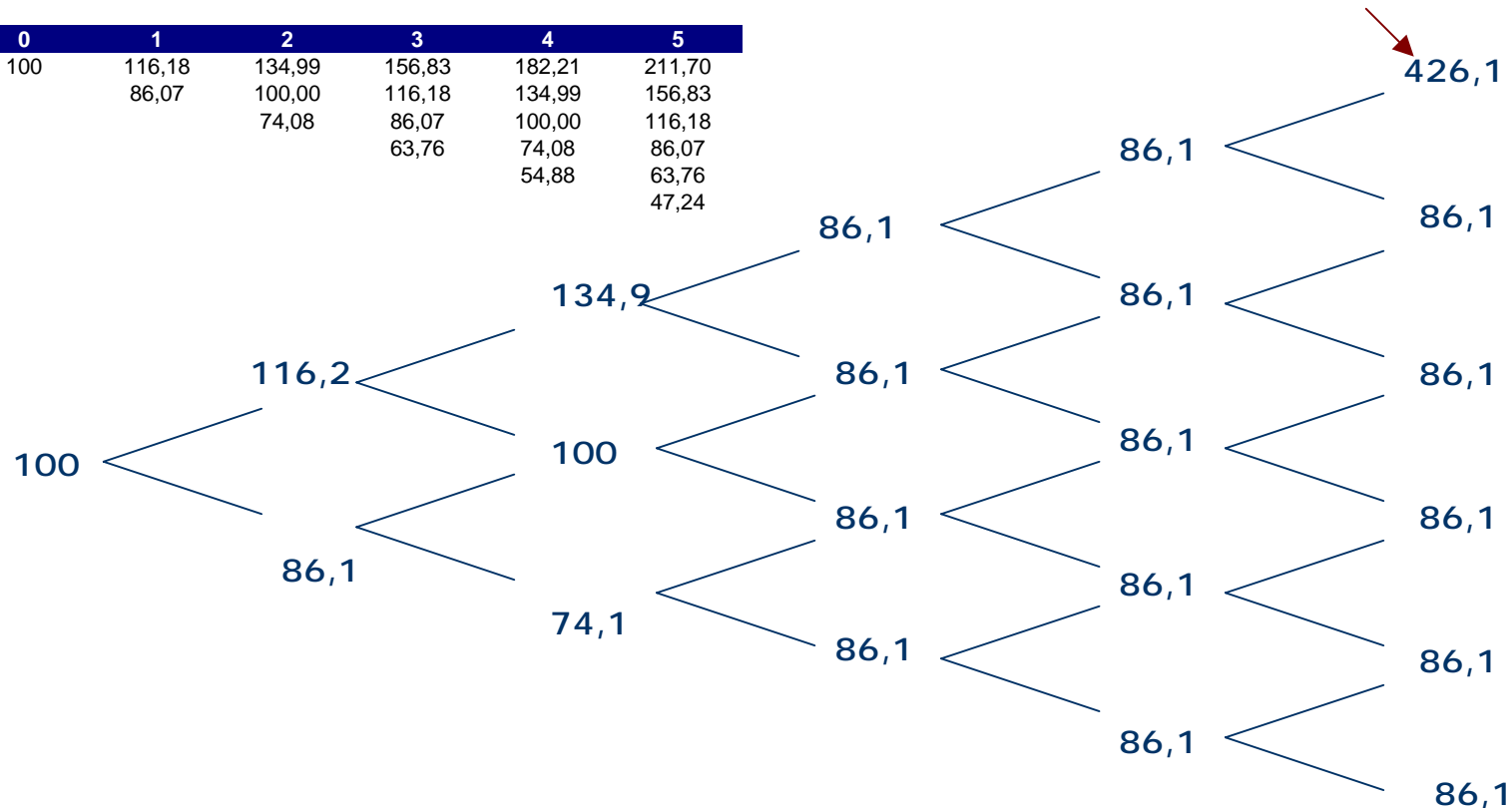
Extension of the option to choose



Now, suppose the same options described in the last example, but with a “twist”. For instance, the expansion factor increases at a 10% rate per year, while the cost of expanding decreases at a 3% per year. Similarly, the savings projected from contracting will reduce at a 10% rate and the value of abandoning increases at a 5% rate.

$$\text{MAX}[211,7 \times 1,3 \times (1,1)^5 - 20 \times (0,97)^5; 211,7(0,9) + 25(0,95)^5; 100(0,95)^5; 211,7]$$

	0	1	2	3	4	5
0	100	116,18	134,99	156,83	182,21	211,70
1		86,07	100,00	116,18	134,99	156,83
2			74,08	86,07	100,00	116,18
3				63,76	74,08	86,07
4					54,88	63,76
5						47,24



Questions



1. Using the example on the abandonment option, recalculate the value of the option assuming that the salvage value increasing 10% at every period from the starting point.
2. Using the example on the expansion option, assumes that the competitor has the same level of uncertainty as the firm being valued. Describe has to be done differently if the competitor is assumed to be growing at a different rate and facing a different set of risks and uncertainties. Rerun the analysis assuming that the competitor's volatility is 45% instead of 35%.
3. Using the example on the simultaneous compound option, but changing the first phase cost to \$500 and the second phase cost to \$900. Should the results comparable? Why or why not?